SET - 2 **GR 14** CODE: GR14A1001

I B. Tech I Semester Supplementary Examinations, June, 2015
Linear Algebra and Single Variable Calculus
(Common to CE, EEE, ME, ECE, CSE, BME and IT)

Time: 3 hours **Max Marks: 70**

PART – A
Answer ALL questions
All questions carry equal marks

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10 * 2 marks = 20 Marks

1. a Find the rank of the matrix $A=\begin{pmatrix}2 & 3 & 1 & -2\\ 1 & 2 & 0 & 2\\ 1 & 4 & -2 & 14\end{pmatrix}$ **[2]**

b Find the ratio of largest to the smallest Eigen Values of the matrix $A=\begin{pmatrix}5 & 2\\ 2 & 2\end{pmatrix}$ **[2]**

c Find the matrix of singular values of the matrix $A=\begin{pmatrix}3 & -1\\ 2 & 4\end{pmatrix}$ **[2]**

d Determine the nature of the quadratic form $Q(X)=17x_1^{\,2}-30x_1\,x_2+17x_3^{\,2}$ **[2]**

e Find *'c'* of the Lagrange's Mean Value Theorem for the function $f(x)=\ln x$ in the interval (e,e^2) correct to 2 decimal places. **[2]**

f Construct the Maclaurin's Series for the function $f(x)=\ln\sqrt{\dfrac{1+x}{1-x}}$ **[2]**

g Find the differential equation of the family of circles with centers on the *y-axis* and passing through the origin. **[2]**

h Find an integration factor of the differential equation $\left(y^4+2y\right)dx+\left(xy^3+2y^4-4x\right)dy=0$ **[2]**

i Find the differential equation of the Orthogonal Trajectories of the family of Strophoids $r=a(\sec\theta+\tan\theta)$ **[2]**

j Find the particular integral of the differential equation $y''+y'+2y=x^2+1$ **[2]**

PART – B

Answer any FIVE questions. All questions carry equal marks

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5 * 10 marks = 50 Marks

2. Find the Moore-Penrose pseudo-inverse of the matrix $A=\begin{pmatrix}1 & 2\\ 2 & 1\\ 1 & -1\end{pmatrix}$. Use this result to obtain the least squares approximate solution of the over determined system $\begin{pmatrix}1 & 2\\ 2 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}5\\ 4\\ -1\end{pmatrix}$ **[10]**

3. Perform a QR factorization of the matrix $A=\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ -1 & 1 & 0\\ 1 & 5 & 1\end{pmatrix}$ using the Gram Schmidt process. **[10]**

4. (a) Verify Cauchy's Mean Value Theorem for the function pair $f(x)=e^{2x}$ *and* $g(x)=e^{-2x}$ defined over the interval $(1,4)$. [4] **[10]**

(b) Verify the Taylor's Theorem with Lagrange's form of remainder for the function $f(x)=(2+x)^{7/3}$ in the interval (0, 1) up to 2 terms. [6]

5. (a) Solve the initial value problem $2xy\,dx+(y^2-x^2)dy=0$ subject to $y(2)=1$ [6] **[10]**

(b) A certain Radioactive Substance 500 g initially, drops its mass to 350 g in 50 years. What is its half life? When does 90 % of its mass disappear? [4]

6. Solve the Cauchy's Equation $x^3y'''+3x^2y''+xy'+8y=260\sin(\ln x)$ **[10]**

7. (a) Prove that the Eigen Values of a Hermitian Matrix are purely real. [4] **[10]**

(b) Compute the matrix $\cos(A)$ for the matrix $A=\begin{pmatrix}1 & 2\\ 3 & 2\end{pmatrix}$. Use 2 decimal approximations. [6]

8. (a) Diagonalize the matrix $A=\begin{pmatrix}7 & -2 & 1\\ -2 & 10 & -2\\ 1 & -2 & 7\end{pmatrix}$ [6] **[10]**

(b) A particle performing simple harmonic motion about a point O is located at distances of 5 cm, 10 cm and 15 cm on the same side of O at $t=2,4$ *and* 6 sec. Determine the time period of oscillation. [4]

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